$$= 4.60 \begin{bmatrix} 1 \\ 0.60 \end{bmatrix}$$

$$\lambda_8 = 4.60, \quad X^8 = \begin{bmatrix} 1 \\ 0.60 \end{bmatrix}$$

Illy $X^8 = \begin{bmatrix} 1 \\ 0.60 \end{bmatrix}$

From above its clear that $X^8 - X^7 = 0$

Hence $\begin{bmatrix} 1 \\ 0.60 \end{bmatrix}$ are the required eigen vectors

& $\lambda = 4.60$ is the required eigen value of given matrix.

Ques $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix}$ & $X = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$$Y^1 = AX = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ -2 \\ 7 \end{bmatrix} = 7 \begin{bmatrix} 0.85 \\ -0.28 \\ 1 \end{bmatrix}$$

$$\lambda_2 = 7 \text{ \& } X^2 = \begin{bmatrix} 0.85 \\ -0.28 \\ 1 \end{bmatrix}$$

→ also $Y^2 = AX^2 = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} 0.85 \\ -0.28 \\ 1 \end{bmatrix} = \begin{bmatrix} 2.73 \\ 3.12 \\ 7 \end{bmatrix} = 7 \begin{bmatrix} 0.39 \\ 0.44 \\ 1 \end{bmatrix}$

$$\lambda_3 = 7 \text{ \& } X^3 = \begin{bmatrix} 0.39 \\ 0.44 \\ 1 \end{bmatrix}$$

→ $Y^3 = AX^3 = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} 0.39 \\ 0.44 \\ 1 \end{bmatrix} = \begin{bmatrix} 4.27 \\ 0.24 \\ 7 \end{bmatrix} = 7 \begin{bmatrix} 0.61 \\ 0.03 \\ 1 \end{bmatrix}$

$$\lambda_4 = 7 \text{ \& } X^4 = \begin{bmatrix} 0.61 \\ 0.03 \\ 1 \end{bmatrix}$$

→ $Y^4 = AX^4 = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} 0.61 \\ 0.03 \\ 1 \end{bmatrix} = \begin{bmatrix} 3.67 \\ 1.86 \\ 7 \end{bmatrix} = 7 \begin{bmatrix} 0.52 \\ 0.26 \\ 1 \end{bmatrix}$

$$\lambda_5 = 7 \text{ \& } X^5 = \begin{bmatrix} 0.52 \\ 0.26 \\ 1 \end{bmatrix}$$

$$Y^5 = AX^5 = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} 0.52 \\ 0.26 \\ 1 \end{bmatrix} = \begin{bmatrix} 3.64 \\ 0.96 \\ 7 \end{bmatrix} = 7 \begin{bmatrix} 0.5 \\ 0.13 \\ 1 \end{bmatrix}$$

$$\lambda_6 = 7 \text{ \& } X^6 = \begin{bmatrix} 0.5 \\ 0.13 \\ 1 \end{bmatrix}$$

→ $Y^6 = AX^6 = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} 0.5 \\ 0.13 \\ 1 \end{bmatrix} = \begin{bmatrix} 3.76 \\ 1.48 \\ 7 \end{bmatrix} = 7 \begin{bmatrix} 0.53 \\ 0.21 \\ 1 \end{bmatrix}$

$$\lambda_7 = 7 \text{ \& } X^7 = \begin{bmatrix} 0.53 \\ 0.21 \\ 1 \end{bmatrix}$$

→ $Y^7 = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} 0.53 \\ 0.21 \\ 1 \end{bmatrix} = \begin{bmatrix} 3.95 \\ 1.16 \\ 7 \end{bmatrix} = 7 \begin{bmatrix} 0.5 \\ 0.16 \\ 1 \end{bmatrix}$

$$\lambda_8 = 7 \text{ \& } X^8 = \begin{bmatrix} 0.5 \\ 0.16 \\ 1 \end{bmatrix}$$

→ $Y^8 = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} 0.5 \\ 0.16 \\ 1 \end{bmatrix} = \begin{bmatrix} 3.82 \\ 1.36 \\ 7 \end{bmatrix} = 7 \begin{bmatrix} 0.54 \\ 0.19 \\ 1 \end{bmatrix}$

→ $Y^9 = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} 0.54 \\ 0.19 \\ 1 \end{bmatrix} = \begin{bmatrix} 3.92 \\ 1.24 \\ 7 \end{bmatrix} = 7 \begin{bmatrix} 0.56 \\ 0.17 \\ 1 \end{bmatrix}$

→ $Y^{10} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} 0.56 \\ 0.17 \\ 1 \end{bmatrix} = \begin{bmatrix} 3.9 \\ 1.32 \\ 7 \end{bmatrix} = 7 \begin{bmatrix} 0.55 \\ 0.18 \\ 1 \end{bmatrix}$

$\rightarrow y^{11} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} 0.55 \\ 0.18 \\ 1 \end{bmatrix} = \begin{bmatrix} 3.91 \\ 1.28 \\ 7 \end{bmatrix} = 7 \begin{bmatrix} 0.55 \\ 0.18 \\ 1 \end{bmatrix}$

$\lambda_{12} = 7$ & $x^{12} = \begin{bmatrix} 0.55 \\ 0.18 \\ 1 \end{bmatrix}$ from the above it is clear that $x^{12} - x^{11} = 0$

Hence $\begin{bmatrix} 0.55 \\ 0.18 \\ 1 \end{bmatrix}$ are required sol^n

& $\lambda = 7$ is the ~~eigen~~ required eigen vector of given matrix

Ques find the eigen value & eigen vector of the matrix $A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ w.r.to the eigen vector $x = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$.

☆ <u>Inverse Power Method</u> This method is also known as "ittiration method" which is used to calculate the min. eigen value & min eigen vectors of the given matrix.

→ let A is the matrix whose inverse can be calculated by the formula.

$$A^{-1} = \frac{1}{|A|} \operatorname{adj} A$$

Then the power method can be represented as $A^{-1}x = \lambda x$

The process (ittiration) is repeated untill $x^{(r)} - x^{1} = 0$

Then the obtained eigen value is represented by $1/\lambda_1$ which give its min. value.

Ques- find the min. eigen value of the given matrix $A = \begin{bmatrix} 3 & 0 & 1 \\ 0 & -3 & 0 \\ 1 & 0 & 3 \end{bmatrix}$ with the eigen vector $x \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$

sol^n $|A| = 3[-9] + 1[3]$

$= -27 + 3 = -24$

$c_{11} = \begin{vmatrix} -3 & 0 \\ 0 & 3 \end{vmatrix} = -9$

$c_{12} = \begin{vmatrix} 0 & 0 \\ 1 & 3 \end{vmatrix} = 0$, $c_{13} = \begin{vmatrix} 0 & -3 \\ 1 & 0 \end{vmatrix} = 3$

$c_{21} = -\begin{vmatrix} 0 & 1 \\ 0 & 3 \end{vmatrix} = 0$, $c_{22} = \begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix} = 8$

$c_{23} = -\begin{vmatrix} 3 & 0 \\ 1 & 0 \end{vmatrix} = 0$, $c_{31} = \begin{vmatrix} 0 & 1 \\ -3 & 0 \end{vmatrix} = 3$

$c_{32} = -\begin{vmatrix} 3 & 1 \\ 0 & 0 \end{vmatrix} = 0$, $c_{33} = \begin{vmatrix} 3 & 0 \\ 0 & -3 \end{vmatrix} = -9$

$y^{1} = A^{-1} x^{1-1}$

$\operatorname{adj} A = \begin{bmatrix} -9 & 0 & 3 \\ 0 & 8 & 0 \\ 3 & 0 & -9 \end{bmatrix}$

$A^{-1} = \begin{vmatrix} 3/8 & 0 & -1/8 \\ 0 & 1/3 & 0 \\ -1/8 & 0 & -3/8 \end{vmatrix}$

Now $A^{-1}x = \begin{bmatrix} 0.37 & 0 & -0.12 \\ 0 & 0.33 & 0 \\ 0.12 & 0 & -0.37 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$